RUTHERFORD SCATTERING WITH RETARDATION

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Numerical solutions for Sommerfeld model in nonrelativistic case are presented for the scattering of a spinless extended charged body in a static Coulomb field of a fixed point charge. It is shown that differential cross section for extended body preserves the form of the Rutherford result with multiplier, not equal to one (as in classical case), but depending on the size of Sommerfeld particle. Also the effect of capture by attractive center is found out for Sommerfeld particle. The origin of this effect lies in radiation damping.
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Here we continue [1] our numerical investigation of Sommerfeld model in classical electrodynamics. Let us remind that Sommerfeld model of charged rigid sphere [2] is the simplest model to take into consideration the "back-reaction" of self-electromagnetic field of a radiating extended charged body on its equation of motion (in the limit of zero body's size we have the known Lorentz-Dirac equation with all its problems: renormalization of mass, preacceleration, runaway solutions, etc.).

In the previous article the effect of classical tunneling was considered - due to retardation moving body begins "to feel" the existence of potential barrier too late, when this barrier is overcome ([1], see also [3]).

Consequently one should expect that Rutherford scattering of a charged extended body in the static Coulomb field of a fixed point charge also differs from classical scattering of point-like particle (for Lorentz-Dirac equation Rutherford scattering was numerically investigated in [4]).

For the case of simplicity here we consider the nonrelativistic, linear in velocity, version of Sommerfeld model.

Let the total charge of a uniformly charged sphere be Q, mechanical mass - m, radius - a. Then its equation of motion reads:

$$m\dot{\vec{v}} = \vec{F}_{ext} + \eta\,[\vec{v}(t-2a/c) - \vec{v}(t)] \tag{1}$$

here $\eta = \frac{Q^2}{3ca^2}$, $\vec{v} = d\vec{R}/dt$, $\vec{R}$ - coordinate of the center of the shell.

External force $\vec{F}_{ext}$, produced by fixed point charge e (placed at $\vec{r} = 0$), is

$$\vec{F}_{ext} = \int d\vec{r}\rho \cdot \frac{e\vec{r}}{r^3}$$

and for

$$\rho = Q\delta(|\vec{r} - \vec{R}| - a)/4\pi a^2$$

reads

$$\vec{F}_{ext} = \frac{e\vec{R}}{R^3}, \quad R > a \tag{2}$$

In dimensionless variables $\vec{R} = \vec{\Pi} \cdot 2L, \quad ct = x \cdot 2L$ equation (1-2) takes the form

$$\ddot{\vec{\Pi}} = K\left[\dot{\vec{\Pi}}(x - \delta) - \dot{\vec{\Pi}}(x)\right] + \lambda \cdot \vec{\Pi} \cdot |\vec{\Pi}|^{-3} \tag{3}$$

with

$$K = \frac{2Q^2L}{3mc^2a^2}, \quad \lambda = \frac{eQ}{2mc^2L}, \quad \delta = a/L$$

or

$$K = \frac{2r_{cl}L}{3a^2}, \quad \lambda = \frac{er_{cl}}{Q2L}, \quad r_{cl} = \frac{Q^2}{mc^2}$$

Taking the $X - Y$ plane to be the plane of scattering ($\vec{\Pi} = (X, Y, 0)$), we split equation (3) into two:

$$\ddot{Y} = K\left[\dot{Y}(x - \delta) - \dot{Y}(x)\right] + \lambda \cdot Y \cdot (X^2 + Y^2)^{-3/2}$$

$$\ddot{X} = K\left[\dot{X}(x - \delta) - \dot{X}(x)\right] + \lambda \cdot X \cdot (X^2 + Y^2)^{-3/2} \tag{4}$$

The starting conditions at $x = 0$ are:

$$X_i = 1000, \quad Y_i = b \ \ (b - impact\ parameter), \quad \dot{X}_i = v_i = -0.1, \quad \dot{Y}_i = 0 \tag{5}$$

Numerical results are expressed on Figs. 1,2,3.

1.

On Fig. 1. one can see how the scattering angle varies from point-like particle (classical scattering, curve 1) to extended body (curve 2). Here we have chosen

$$L = 5r_{cl}, \quad b = 60.0, \quad \delta = 4.0, \quad \lambda = 0.1, \quad K = (2/15)(\delta)^{-2}$$

i.e.

$$a = 20r_{cl}, \quad e = Q, \quad K = (10/3)(r_{cl}/a)^2 = 1/120$$

vertical axis is Y, horizontal - X.

Thus due to retardation the scattering angle θ for extended body is smaller than that for point-like particle.

2.

Differential cross section $d\sigma$ is given by the formula

$$d\sigma = 2\pi\rho(\theta)|\frac{d\rho(\theta)}{d\theta}|d\theta$$

where $\rho = b \cdot 2L$, or

$$\frac{1}{2\pi(2L)^2} \cdot \frac{d\sigma}{d\xi} = \frac{db^2}{d\xi} \tag{6}$$

where

$$\xi = \frac{1+\cos\theta}{1-\cos\theta}$$

Classical Rutherford result is that R.H.S. of eq. (4) is constant:

$$b^2 \cdot (v_i)^4 \cdot (\lambda)^{-2} = \xi \tag{7}$$

or

$$\frac{(\lambda)^2}{2\pi(2L)^2(v_i)^4} \cdot \frac{d\sigma}{d\xi} = 1 \tag{8}$$

This classical result can be derived from eq.(4) in standard manner for $K = 0$ (see, for ex., [5])

In the case of extended body

$$L = 5r_{cl}, \quad \lambda = 0.1, \quad K = (2/15)(\delta)^{-2}$$

i.e.

$$e = Q, \quad K = (10/3)(r_{cl}/a)^2$$

numerical calculations for various values of b, $10.0 < b < 110.0$ show that Rutherford formula (7,8) changes in the following way:

$$b^2 \cdot (v_i)^4 \cdot (\lambda)^{-2} = \xi \cdot [1 + const/\delta]^{-1} \tag{9}$$

or

$$\frac{(\lambda)^2}{2\pi(2L)^2(v_i)^4} \cdot \frac{d\sigma}{d\xi} = [1 + const/\delta]^{-1} \tag{10}$$

where the multiplier *const* is equal approximately to 0.30.

Thus differential cross section for extended body preserves the form of the Rutherford result with multiplier, not equal to one (as in classical case), but depending on the value of size of Sommerfeld particle. For $\delta \to \infty$ (i.e. $K \to 0$) formula (9,10) gives the Rutherford result.

On Fig. 2 we see how the direct proportionality between $b^2 \cdot (v_i)^4 \cdot (\lambda)^{-2}$ and ξ changes in accordance to formula (9). Vertical axis is $b^2 \cdot (v_i)^4 \cdot (\lambda)^{-2}$ and horizontal - ξ. Values of retardation δ (or dimensionless body's size) are taken to be 1, 2, 3, 4, and curves are marked accordingly as 1, 2, 3, 4; in the case of Rutherford scattering ($K \equiv 0$) the curve is marked as "R".

3.

On Fig.3 we see the appearance of the effect of capture of Sommerfeld particle with charge Q by the attractive Coulomb center with charge e. Here we have chosen the following values of parameters:

$$L = r_{cl}/2, \quad \lambda = -1.0, \quad K = (4/3)(\delta)^{-2}, \quad \delta = 5.0, \quad b = 30.0$$

i.e.

$$e = -Q, \quad a = 2.5 r_{cl}, \quad K = 4/75.$$

Initial conditions are the same as in (5).

Following classical Rutherford scattering (eq. (4) with $K \equiv 0$) for initial conditions (5) trajectory must be infinite one and thus there is no capture; but this is not the case of Sommerfeld particle: due to radiation damping particle loses its energy and consequently can fall down on the attractive center.

Varying impact parameter b with fixed $\lambda = -1.0$ and $\delta = 5.0$ we numerically found out the crucial value of b when the effect of capture begins:

$$b \leq b_{cr} \approx 31.40$$

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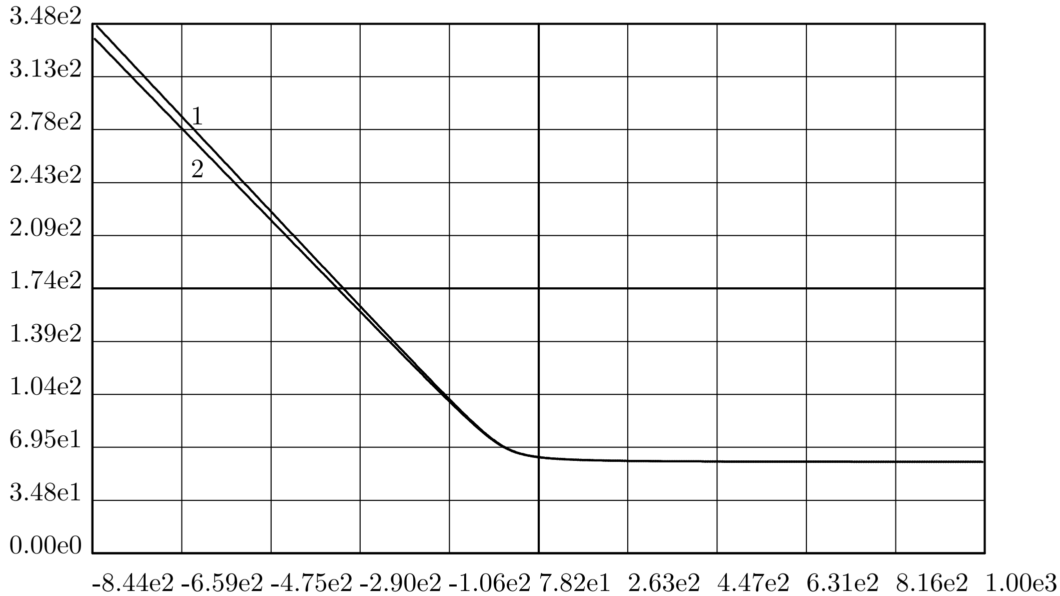
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Fig. 1

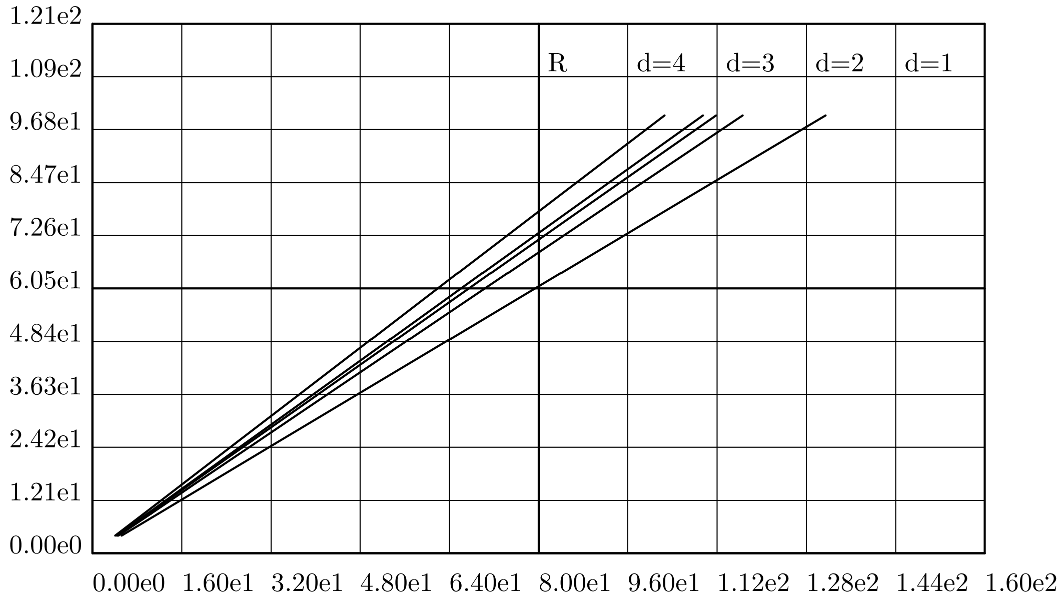
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Fig. 2

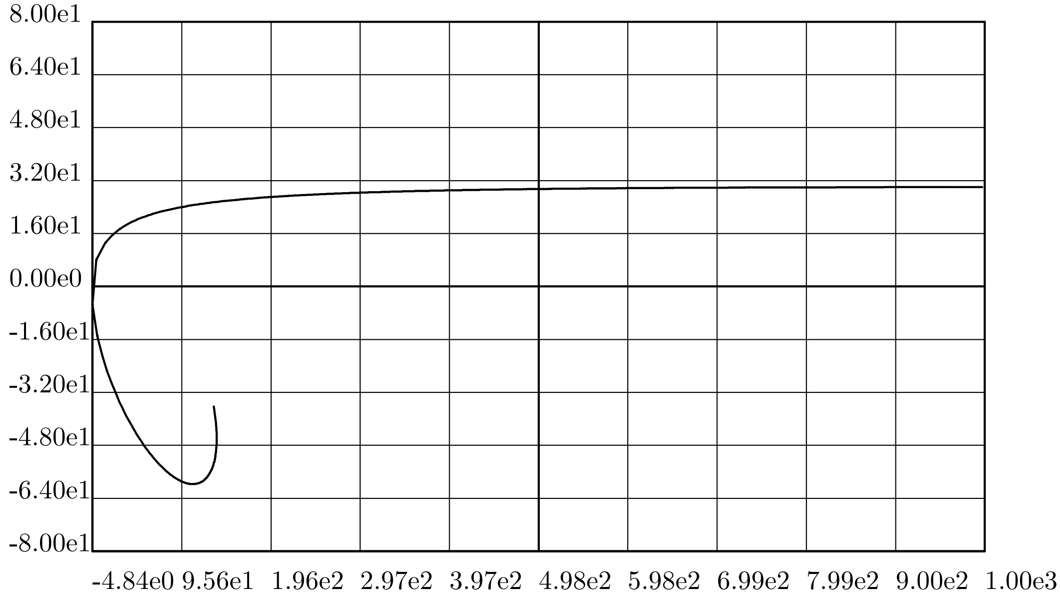
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Fig. 3